

Music and Maths.

(Imagen taken from <http://motivate.maths.org/conferences/conference.php?conf_id=35>)

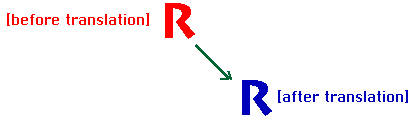
1. Defining "sound" as a phenomenon  
 We considered sound as everything that can be perceived by the animal or human hearing. Sound are waves, sound is a physical phenomenon that needs a physical medium to exist: sound can be transmitted through liquids, through gases, even through solids, but it can not be transmitted through emptiness. It also needs something to produce it and someone or something to receive it (a receptor). There are many machines created by man that receive sound, analyze it or record it, so this receptor is not necessarily an alive been.

2. Defining "music" and "noise".  
 Sounds can be combined anyway. Usually, when you combined sounds following an order, if the arrangement results to be pleasant to human hearing, then we talk about a musical set. Music is defined as an artistic creation in which an arrangement of sounds impresses pleasantly human hearing.  
 If many sounds are produced in a disordered way and the set results to be displeasing to human earing, then we talk about noise  
 So, either noise or music are kind of subjective concepts, because they depend on the receptor; you might think rock is noisy, but a lot of people like it and classify it as good music.

3. First discovery about music and mathematics: Pythagorean Society and The Golden Ratio.

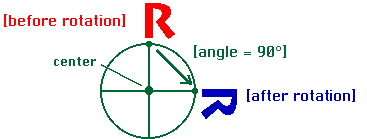
First discovery about music and Maths were made hundreds of years b.C. by Pythagorean Society. It is said that Pythagoras took a string and produce a sound with it, and then he started looking for fractions of that strings which sound he could combine with the first one to produce music. He tried a lot until he found out there were many pieces of the string that produced sound that could be pleasantly combined we the original size sound... and all those pieces were fractions of the string according to several applications of the Golden Ratio. He found out that applying this ratio, no matter how many times, all those pieces of string produced sounds that combined -no matter the order- resulted into a musical arrangement. So, artistic creation and maths were related; beauty and numbers were the same.  
  
4. Mathematical expression of musical scale: Is there equivalence between mathematical formulae and musical scores?  
 This first relationship between golden ratio and music turned into the idea you must be able to express music using numbers. Well, it turns out that each sound is identified with a musical note, and each musical note is associated to its frequency, which is the number of times the wave of sound of that note vibrates on air during a second. Of course, that frequency is measured with a number. So, you do can assign a number to each note, and so describe musical arrangements trough sequences of numbers, but this "equivalence" is not as useful as you might think. You might translate sets of numbers into notes, or random notes into numbers, but, there is no way to translate math, physics or any other scientific real formula into a musical arrangement and there is no way to translate musical arrangements into any real useful formula. Therefore, it is just a formal, structural equivalence; it does not have scientific applications.

5. Geometrical rigid transformations: translation, rotation and symmetries.   
Another nice tip about relation between music and Math is related to plane geometry, which is the branch of Math that studies all the elements that can be drawn on a plane. Movements that take any element of the plane, like point, a straight line, a circumference or any other, and change its position or it orientation without changing it size or form are called rigid transformations. There are basically four rigid transformations:

**Translations:**

These are movements that slide a figure from its original place to any other, respecting its orientation in the plane.

 (Image taken from [http://mathforum.org/sum95/suzanne/symsusan.html](http://www.learner.org/courses/learningmath/geometry/session7/part_c/index.html))

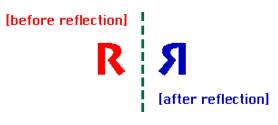
**Rotation:**

This movement takes an object and a picks point in plane, and moves the object describing an angle, as if it were on a circumference centered on the selected point, changing its position and respecting its orientations.

 (Image taken from [http://mathforum.org/sum95/suzanne/symsusan.html](http://www.learner.org/courses/learningmath/geometry/session7/part_c/index.html))

**Symmetries:**

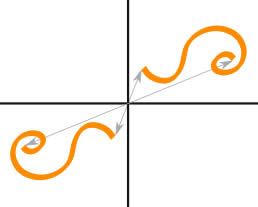
These are movements that change the position and the orientation of the figures. There are two types of symmetries :



**Axial symmetry:**

This movement takes a figure and picks a straight line, and transforms the figure into its opposite image at the other side of the line, preserving the distances, just as if you were looking its image through a mirror. This movement is also called **reflection.**

 (Image taken from [http://mathforum.org/sum95/suzanne/symsusan.html](http://www.learner.org/courses/learningmath/geometry/session7/part_c/index.html))

**Point symmetry:**

This movement takes a figure and a point on the plane, and produces its image reflecting it against two perpendicular lines through the selected point. This movement reverse it position and its orientation vertically and horizontally, so it is also known as a second order reflection.

 (Image taken from <http://www.mathsisfun.com/geometry/symmetry.html>)

6. Composing musical patterns using rigid transformations.

A simple and interesting relationship between geometry and music comes out from rigid transformations. It turns out that whenever you get a musical arrangement and you write it down on a score and you transform it trough a rigid transformation, you can combined the original set and the transformed one anyway, and the result continues being a pleasant arrangement .And it is true if you do this one or many times.



Applying translations to a set of notes means repeating the set.

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Reflecting a set of notes turns into a new composition which is called the retrograde score.

You can find this type of composition in the song “Rain drops keep falling in my head”, by B.J. Thomas.

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Applying point symmetry to a set of notes results is also useful at composing. You can find this idea in the traditional English song “Greensleeves”.**

7. Applying probabilities at musical composition: Mozart’s dice game.

There are some random ways to compose music. Probabilities can be use to design a musical arrangement following a guide. It works associating a note to every value the variable could take. One way to do it was produced by Mozart, who designed a chart that associates notes to values you can get rolling two dices eight times, creating a short pattern. Then repeating the process, it creates a second patters, and again, and again, until creating a long score. Well, it turns out that this method produce billions of different compositions! And the chart designed by Mozart guarantees the composition will be musical, not noisy, no matter the order you take and combine the patterns you produce each time.